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NOTES ON GRAIN PRESSURES IN STORAGE BINS.

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CONTENTS.

INTRODUCTION.

The formula derived by Janssen for the determination of the vertical and lateral pressures of grain in bins is one commonly used by grain elevator engineers. Most engineers, in their calculations, have also made use of a table of values similar to the one here given, by means of which the unit pressures are more quickly and easily determined (see Table 1). In this bulletin are given rules for the more advantageous application of this table, together with additional tables and graphical charts which should further facilitate the designing of grain bins.

JANSSEN'S FORMULA.[1]

In Janssen's formula the following nomenclature is used:

A = area of bin in square feet.

P = perimeter of bin in feet.

R = hydraulic radius of bin $= \frac{A}{P}$

NOTE.—This bulletin is of interest to engineers, particularly those having occasion to design grain storage bins.

[1] For the derivation of Janssen's formula together with descriptions and results of different experiments to determine values for the coefficient of friction of grain on bin walls and the ratio of the lateral to the vertical pressure at any point, see Ketchum, Milo S., The design of walls, bins, and grain elevators, 2d ed., New York, 1911.

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D = diameter of bin or diameter of inscribed circle in feet.
H = height in feet of grain above point in question.
μ' = coefficient of friction of grain on bin walls.
W = weight of grain in pounds per cubic foot.
V = vertical pressure of grain at depth H in pounds per square foot.
L = lateral pressure of grain at depth H in pounds per square foot.
$k = \frac{L}{V}$ = ratio of lateral to vertical pressure.
e = base of Naperian logarithms.

NOTE.—Both V and L are assumed to be constant at all points on a horizontal plane.

$$V = \frac{R \times W}{k \times \mu'}\left(1 - \frac{1}{e^{\frac{k \times \mu' \times H}{R}}}\right)$$

For round and for square bins or other bins of the form of a regular polygon, the hydraulic radius $\frac{A}{P}$ is equal to $\frac{D}{4}$ and the formula becomes

$$V = \frac{D \times W}{4 \times k \times \mu'}\left(1 - \frac{1}{e^{\frac{4 \times k \times \mu' H}{D}}}\right)$$

$$L = kV$$

VALUES OF μ' AND k.

Numerous experiments have been made with various grains in bins constructed of different materials to determine values for the factors μ' and k. The results of several of these experiments are given in the appendix.

PRESSURE FACTORS FOR WHEAT IN ROUND OR OTHER REGULAR CONCRETE BINS.

By taking μ' equal to 0.4167, an average value for wheat on concrete, and k equal to 0.6, an accepted value for wheat in concrete bins, the product $4 \times k \times \mu'$ becomes equal to 1.0, and with W equal to 50 pounds, the formula simplifies to

$$V = 50 \times D\left(1 - \frac{1}{e^{\frac{H}{D}}}\right)$$

$$\text{or } \frac{V}{D} = 50 \quad \left(1 - \frac{1}{e^{\frac{H}{D}}}\right)$$

$$\text{and } \frac{L}{D} = 0.6\frac{V}{D}$$

Values for the pressure factors $\frac{V}{D}$ and $\frac{L}{D}$ may be tabulated for different values of $\frac{H}{D}$. (See Table 1.)

TABLE 1.—*Pressure factors for wheat in round or other regular concrete bins.*

(Pounds per square foot in diameter.)

$\frac{H}{D}$	$\frac{V}{D}$	$\frac{L}{D}$	$\frac{H}{D}$	$\frac{V}{D}$	$\frac{L}{D}$
0.1	4.76	2.86	2.3	44.97	26.98
.2	9.06	5.44	2.4	45.46	27.28
.3	12.96	7.77	2.5	45.89	27.53
.4	16.48	9.88	2.6	46.29	27.77
.5	19.67	11.80	2.7	46.64	27.98
.6	22.56	13.53	2.8	46.96	28.18
.7	25.17	15.10	2.9	47.25	28.34
.8	27.53	16.52	3.0	47.51	28.50
.9	29.67	17.80	3.1	47.75	28.64
1.0	31.61	18.96	3.2	47.96	28.77
1.1	33.36	20.01	3.3	48.16	28.89
1.2	34.94	20.96	3.4	48.33	28.99
1.3	36.37	21.82	3.5	48.49	29.09
1.4	37.67	22.60	3.6	48.63	29.18
1.5	38.84	23.30	3.7	48.76	29.26
1.6	39.90	23.94	3.8	48.88	29.33
1.7	40.87	24.52	3.9	48.99	29.39
1.8	41.74	25.04	4.0	49.08	29.45
1.9	42.52	25.51	5.0	49.66	29.79
2.0	43.23	25.94	6.0	49.88	29.92
2.1	43.88	26.32	7.0	49.95	29.97
2.2	44.46	26.67	10.0	49.99	29.99

For use of this table see example 1, page 10.

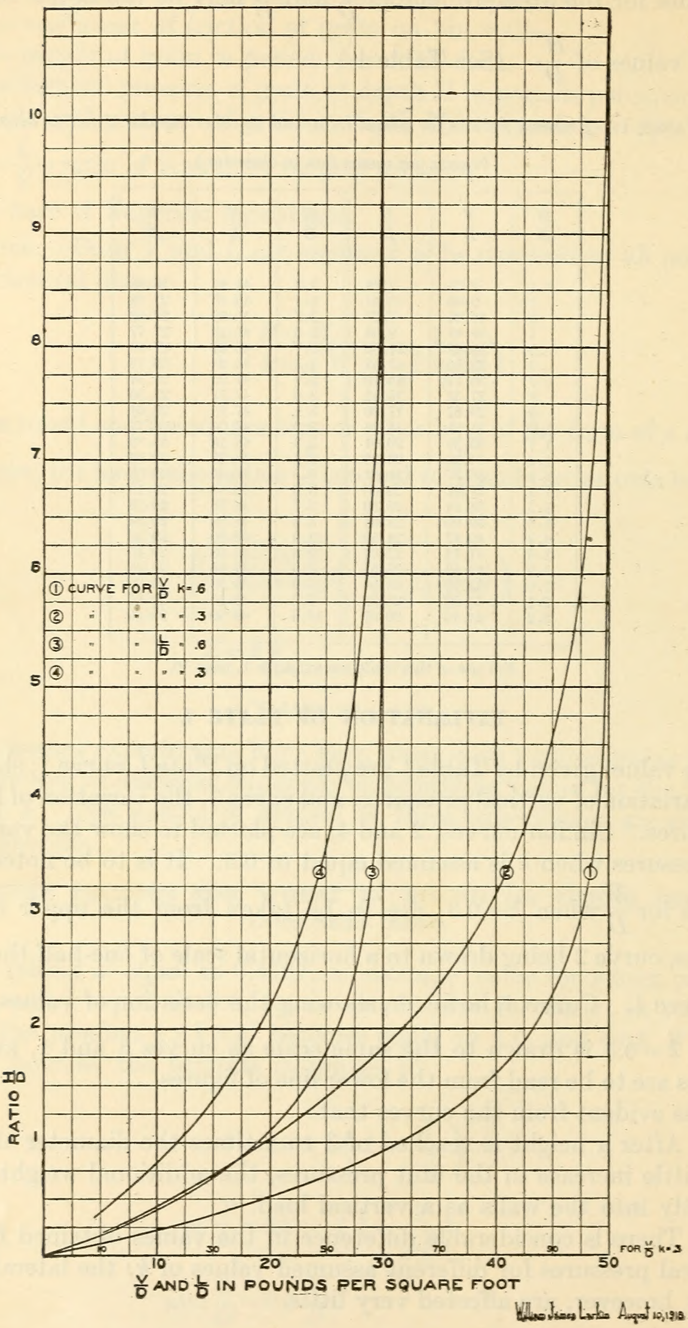
EXPLANATION OF PLATE I.

The values given by Table 1 are plotted on Plate I, curve 1, showing the variation of vertical pressures, and curve 3, the variation of lateral pressures. Similar curves, 2 and 4, are plotted to show the variation of pressures when k is assumed equal to 0.3. It is to be noted that values for $\frac{V}{D}$ when $k=0.3$, are to be taken from the upper line of figures, curve 2 being drawn to a horizontal scale of one-half the scale of curve 1. Curve 4, however, showing the variation of values for $\frac{L}{D}$ when $k=0.3$ is drawn to the same scale as curves 3 and 1, and the values are to be read from the lower line of figures.

It is evident from the curves that:

A. After a height is reached of 3 to 4 times the diameter there is but little increase in the unit pressures, the additional weight going directly into the walls as a vertical load.

B. There is considerable difference in the values obtained for the vertical pressures for different assumed values of k; the lateral pressures, however, are affected very little.


RATIO H/D
① CURVE FOR V/D K=.6
② " " " .3
③ " " L/D .6
④ " " " .3
FOR V/D K=.3
V/D AND L/D IN POUNDS PER SQUARE FOOT
William James Larkin August 10, 1918

(*a*) In very shallow bins there is little difference in the vertical pressures obtained for different values of k, while as the bins become deeper $\frac{V}{D}$ for $k=0.3$ rapidly approaches a value twice as great as for $k=0.6$.

(*b*) In snallow bins the value obtained for the lateral pressures for $k=0.3$ are about one-third less than for $k=0.6$, but as the bins become deeper $\frac{L}{D}$ for $k=0.3$ rapidly approaches the same value as for $k=0.6$.

(*c*) As most grain bins are deep bins, the best general value for k is 0.6. This gives the maximum value for the lateral pressure and also gives the maximum value for the vertical load carried by the bin walls. It is very essential that the determination of the bearing area of the walls and in some cases the design of the foundation, be based on maximum values for the vertical wall loads.

Possibly it would seem best in designing the bin bottom to use the maximum values for the vertical pressure obtained when k is taken equal to 0.3. Most designers, however, work on the assumption that 0.6 is more nearly the correct value for k than 0.3 and use the lesser values for the vertical pressures.

DETERMINATION OF UNIT PRESSURES IN RECTANGULAR OR OTHER IRREGULAR BINS.

Accepting Janssen's formula as a general one applying to all forms of bins, we can conclude that the unit pressures in any bin are equal to those in any other bin of the same hydraulic radius. For round or other regular bins $4 \times R = D$ where R = hydraulic radius. Then the hydraulic radius of any bin multiplied by 4 will give a value equal to the diameter of the equivalent round bin and the pressure factors may be obtained from Table 1.

DETERMINATION OF UNIT PRESSURE FOR VARIOUS GRAINS IN BINS OF DIFFERENT MATERIALS.

Tables 1 and 2 were developed on the assumption that for wheat in concrete bins k equals 0.6 and μ' equals 0.4167. The product $4 \times k \times \mu'$ thus becomes equal to 1 and W was taken equal to 50 pounds.

These tables may be readily used to figure the unit pressures and reinforcing in any bin for any assumed value of k and μ', by computing a compensated value for D which may be designated by D'.

Original formula for regular bins:

$$V = \frac{D \times W}{4 \times k \times \mu'}\left(1 - \frac{1}{e^{\frac{4 \times k \times \mu' \times H}{D}}}\right)$$

If $4 \times k \times \mu' = N$, not equal to 1,

$$V=\frac{D}{N}\times W\left(1-\frac{1}{e^{\frac{H}{\frac{D}{N}}}}\right)$$

Let $\frac{D}{N}=D'$, then the formula becomes,

$$V=D'\times W\left(1-\frac{1}{e^{\frac{H}{D'}}}\right)$$

$$\frac{V}{D'}=W\left(1-\frac{1}{e^{\frac{H}{D'}}}\right)$$

$$L=kV$$

For values of W other than 50 pounds V and L may be determined by direct proportion. (See Example 4, page 14.)

HORIZONTAL REINFORCING FOR CYLINDRICAL CONCRETE BINS.

In a cylindrical tank the total pressure on any diameter for 1 foot in height is equal to the lateral pressure per square foot at that level times the diameter in feet, or $L \times D$. This pressure, tending to burst the tank, produces tension in the walls, and, since two cross sections of the wall are cut by each diameter, the total tensile stress on each cross section of the wall for 1 foot in height is equal to $\frac{L\times D}{2}$. Assuming that all of the tensile stress is taken by the steel reinforcing, and indicating the allowable unit stress in the steel by f_s, the cross-sectional area of steel required in square inches per foot in height at that level is given by the formula:

$$a=\frac{L\times D}{2\times f_s}$$

This formula may be written,

$$\frac{a}{D^2}=\frac{L}{D}\times\frac{1}{2\times f_s}$$

Since the pressure factor $\frac{L}{D}$ is determined by $\frac{H}{D}$, we can tabulate values for the steel factors $\frac{a}{D^2}$ for different values of $\frac{H}{D}$. (See Table 2.)

TABLE 2.—*Steel factors for horizontal reinforcing in cylindrical concrete tanks.*

[Areas of steel per foot in height per diameter squared.]

$\frac{H}{D}$	$\frac{a}{D^2}$			$\frac{H}{D}$	$\frac{a}{D^2}$		
	$f_s=16000$	$f_s=18000$	$f_s=20000$		$f_s=16000$	$f_s=18000$	$f_s=20000$
0.1	0.000089	0.000079	0.000072	2.3	0.000843	0.000749	0.000675
.2	170	151	136	2.4	853	758	682
.3	243	216	194	2.5	860	765	688
.4	309	274	247	2.6	868	771	694
.5	369	328	295	2.7	874	777	700
.6	423	376	338	2.8	881	783	705
.7	472	419	378	2.9	886	787	709
.8	516	459	413	3.0	.000891	.000792	.000713
.9	556	494	445	3.1	895	795	716
1.0	.000593	.000527	.000474	3.2	899	799	719
1.1	625	556	500	3.3	903	802	722
1.2	655	582	524	3.4	906	805	725
1.3	682	606	546	3.5	909	808	727
1.4	706	628	565	3.6	912	810	729
1.5	728	647	583	3.7	914	813	731
1.6	748	665	599	3.8	917	815	733
1.7	760	675	608	3.9	918	816	735
1.8	783	695	626	4.0	.000920	.000818	.000736
1.9	797	708	638	5.0	931	827	745
2.0	.000811	.000720	.000649	6.0	935	831	748
2.1	823	731	658	7.0	936	832	749
2.2	833	741	667	10.0	.000937	.000833	.000750

[For use of this table, see Example 2, page 11.

HORIZONTAL REINFORCING FOR STRAIGHT WALLS OF CONCRETE BINS.

In all bins other than cylindrical tanks the walls are designed as slabs supporting at any given level a uniform load equal to the lateral grain pressure at that point.

Tables 3*a*, 3*b*, and 3*c*, for different allowable stresses in steel and concrete, give the maximum resisting moments and the steel areas required for different effective thicknesses of walls. Also, for different conditions of continuity, constants are listed for determining the heights at which the size or spacing of the bars may be changed.

In the following, the standard nomenclature for reinforced concrete is used also: Let—

L = lateral pressure of the grain in pounds per square foot.

l = span of wall in feet.

t = thickness of wall in inches.

e = thickness of concrete outside of steel in inches.

$d=t-e$ = effective depth to steel in inches.

a = area of steel per foot in height in square inches.

M = bending moment or resisting moment in inch pounds.

m = coefficient of continuity for figuring bending moment.

= 8 for simple spans $M=\frac{L\times l^2\times 12}{8}$

= 10 for end spans—partial continuity $M=\frac{L\times l^2\times 12}{10}$

= 12 for intermediate spans—full continuity $M=\frac{L\times l^2\times 12}{12}$

For any effective depth to steel the maximum resisting moment, which is developed when the steel and concrete are simultaneously stressed to the assumed allowable values for f_s and f_c, is given by the equation:

$$M = f_s jpbd^2$$

For selected values of f_s, f_c, and n and for $b=12$, the product $f_s jpb$ is a constant and

$$M = Cd^2$$

The area of steel required is given by the equation

$$a = pbd$$

The area of steel may also be figured from the equation

$$a = \frac{M}{f_s \times j \times d} \quad \text{or}$$

$$a = \frac{L \times l^2 \times 12}{m \times f_s \times j \times d}$$

For selected values of f_s, f_c, and n and for different values of m and d the product $\frac{m \times f_s \times j \times d}{12}$ is a constant which may be designated by R. Then

$$a = \frac{L \times l^2}{R}$$

$$L = \frac{a \times R}{l^2}$$

The thickness of the wall is determined by the bending moment at the bottom, and, as this thickness is maintained for the full height, it will be greater in the upper portion of the wall than required to develop the assumed stresses. The stress in the concrete will decrease from bottom to top and, as the thickness of the wall is usually selected in even inches, the stress at the bottom will generally be less than the assumed allowable value. In some cases, however, a wall thickness may be chosen slightly less than actually required for the moment at the bottom, in which case the stress in the concrete at that level will exceed the assumed value.

With each change from the bottom to the top in the spacing or size of the reinforcing bars, there is a decrease in the value for the percentage of steel, and, assuming n to be constant, a corresponding increase in the value for j. Therefore, the results obtained by use of the formula $L = \frac{a \times R}{l^2}$, for which R has been computed for a constant value of j, are not absolutely correct. In any case, however, the error is small and, moreover, is on the safe side, because, if the increased value for j was used, slightly larger values for L, the allowable pressure for a given steel area, would be obtained.

TABLE 3A.—*Maximum resisting moments developed and steel areas required for different thicknesses of straight bin walls per foot in height.*

$n=15$

$f_s=16000$ $f_c=650$ $p=0.0077$ $k=0.378$ $j=0.874$					$C=b\,f_s\,p\,j=1292$ $p\,b=0.0924$		
Thickness of wall in inches. t	Concrete outside of steel in inches. e	Depth to steel in inches. d	Maximum resisting moment in inch pounds. $M=1292d^2$	Required steel area in square inches. $a=0.0924d$	Values for $R=\frac{m\,j\,f_s\,d}{12}=1165\,m\,d$		
					$m=8$	$m=10$	$m=12$
4	1	3	11600	0.277	28000	35000	42000
5	1	4	20700	.370	37300	46600	56000
6	1¼	4¾	29200	.439	44300	55300	66400
6	1	5	32300	.462	46600	58300	70000
7	1¼	5¾	42700	.531	53700	67000	80400
7	1	6	46500	.554	56000	70000	83900
8	1½	6½	54600	.600	60600	75700	90900
8	1¼	6¾	58900	.624	62900	78600	94400
9	1½	7½	72700	.693	70000	87400	104900
9	1¼	7¾	77600	.716	72200	90300	108400

TABLE 3B.—*Maximum resisting moments developed and steel areas required for different thicknesses of straight bin walls per foot in height.*

$n=15$

$f_s=18000$ $f_c=700$ $p=0.0072$ $k=0.368$ $j=0.877$					$C=b\,f_s\,p\,j=1364$ $p\,b=0.0864$		
Thickness of wall in inches. t	Concrete outside of steel in inches. e	Depth to steel in inches. d	Maximum resisting moment in inch pounds. $M=1364d^2$	Required steel area in square inches. $a=0.0864d$	Values for $R=\frac{m\,j\,f_s\,d}{12}=1315\,m\,d$		
					$m=8$	$m=10$	$m=12$
4	1	3	12300	0.259	31600	39400	47300
5	1	4	21800	.346	42100	52600	63100
6	1¼	4¾	30800	.410	50000	62500	75000
6	1	5	34100	.432	52600	65800	78900
7	1¼	5¾	45100	.497	60500	75600	90700
7	1	6	49100	.518	63100	78900	94700
8	1½	6½	57600	.561	68400	85500	102600
8	1¼	6¾	62140	.583	71000	88800	106500
9	1½	7½	76700	.648	78900	98600	118400
9	1¼	7¾	81920	.670	81500	101900	122300

TABLE 3C.—*Maximum resisting moments developed and steel areas required for different thicknesses of straight bin walls per foot in height.*

$n=15$

$f_s=20000$ $f_c=700$ $p=0.0060$ $k=0.344$ $j=0.885$					$C=b\,f_s\,p\,j=1274$ $p\,b=0.072$		
Thickness of wall in inches. t	Concrete outside of steel in inches. e	Depth to steel in inches. d	Maximum resisting moment in inch pounds. $M=1274d^2$	Required steel area in square inches. $a=0.072d$	Values for $R=\frac{m\,j\,f_s\,d}{12}=1475\,m\,d$		
					$m=8$	$m=10$	$m=12$
4	1	3	11500	0.216	35400	44300	53100
5	1	4	20400	.288	47200	59000	70800
6	1¼	4¾	28800	.342	56100	70000	84100
6	1	5	31900	.360	59000	73800	88500
7	1¼	5¾	42100	.414	67900	84800	101700
7	1	6	45900	.432	70800	88500	106200
8	1½	6½	53900	.468	76700	95900	115000
8	1¼	6¾	58000	.486	79700	99600	119400
9	1½	7½	71700	.540	88500	110600	132750
9	1¼	7¾	76600	.558	91500	114300	137150

DETERMINATION OF VERTICAL AND LATERAL PRESSURES IN GRAIN BINS WITH HORIZONTAL REINFORCING FOR CYLINDRICAL CONCRETE TANKS BY MEANS OF PLATE II.

By means of the four curves on the chart in Plate II, which have been plotted from values given by Tables 1 and 2, the vertical and lateral unit pressures may be determined for any bin, and in the case of cylindrical concrete tanks the size and spacing of the horizontal reinforcing may be obtained without the need of any calculations.

The manner in which this chart may be used can best be explained by means of several examples, which will be first solved mathematically, using the values given by Tables 1 and 2, and then directly from the chart.

EXAMPLES SHOWING USE OF PLATE II.

EXAMPLE 1.

Required: The vertical and lateral unit pressures at the bottom in a round or square bin 15 feet in diameter and 75 feet in height.

$$\frac{H}{D}=\frac{75}{15}=5$$

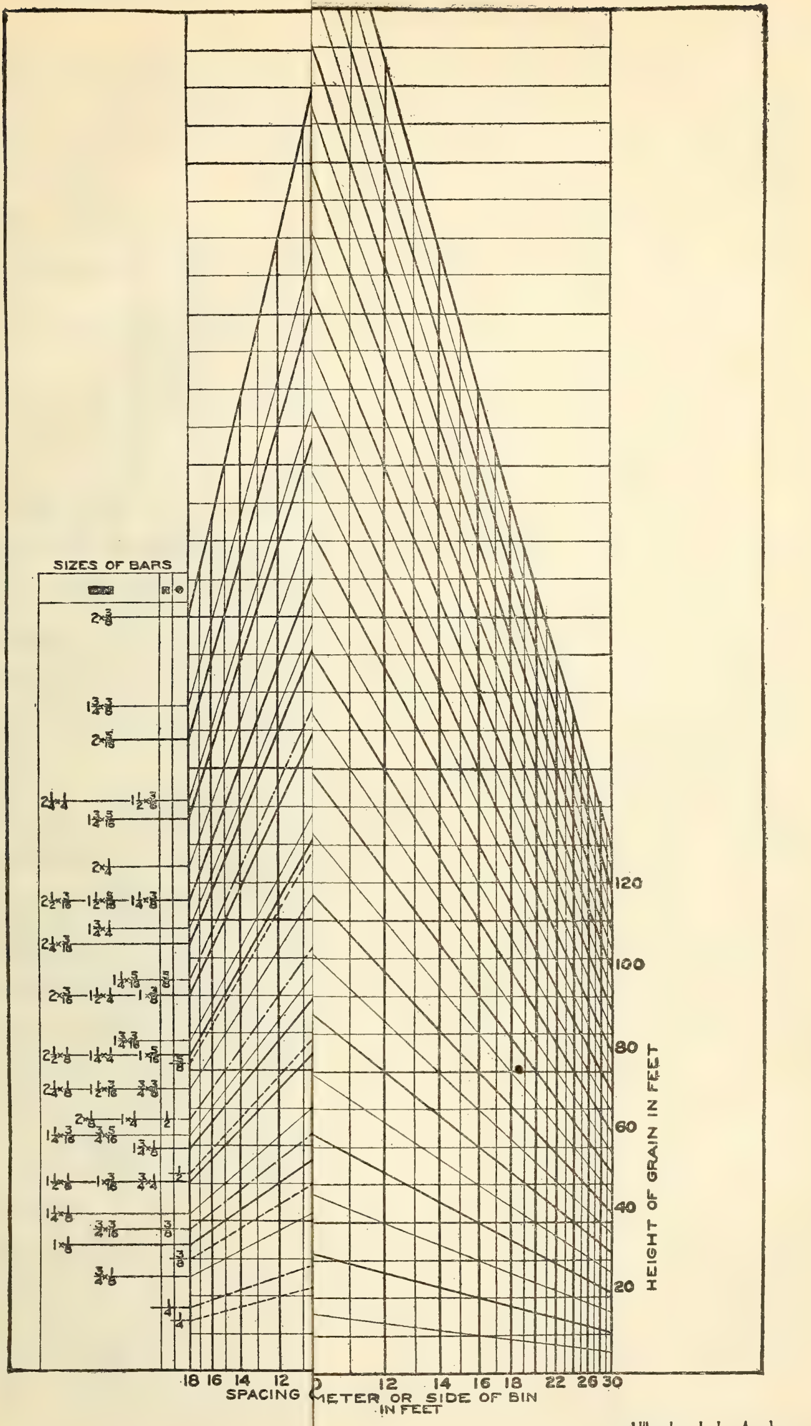
From Table 1: For $\frac{H}{D}=5$

$\frac{V}{D}=49.66$ and $V=15\times 49.66=745$ pounds per square foot.

$\frac{L}{D}=29.79$ and $L=15\times 29.79=447$ pounds per square foot.

From Plate II: Follow the vertical line at the right-hand side for diameter 15 feet upward to its intersection with the oblique line representing 75 feet for the height of the grain. This will be found to be on the heavy horizontal line fourth from the bottom of the page. Follow this horizontal line to the left to its intersection with curve 1. Project from this intersection vertically to the line for diameter 15 feet of the set of radial lines for pressures. This last intersection will be found to be horizontally opposite a point on the scale for pressures a little above the line for 750 or about 745 pounds per square foot, which equals the unit vertical pressure. Returning to the fourth heavy line from the bottom, continue to the left to its intersection with curve 4, again project vertically to radial line 15 for pressures, then horizontally to the right to a little above the line for 450 or about 445 pounds per square foot, which equals the unit lateral pressure.

 PLATE II.



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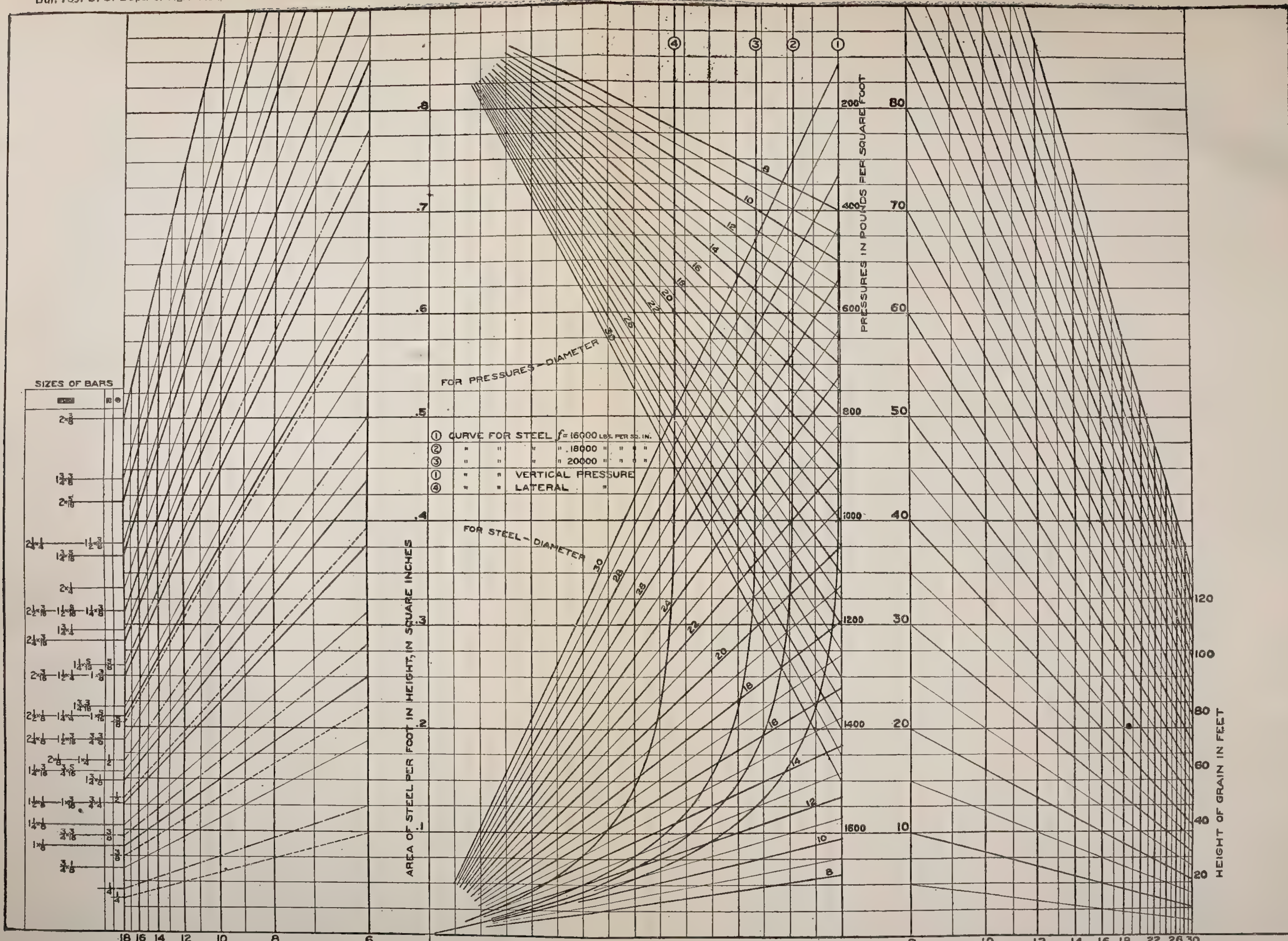

VERTICAL AND LATERAL PRESSURES IN GRAIN BINS
WITH
HORIZONTAL REINFORCING FOR CYLINDRICAL CONCRETE TANKS

William James Larkin August 10, 1916.

EXAMPLE 2.

Required: The area of steel in square inches per foot in height for horizontal reinforcing for a cylindrical concrete tank 25 feet in diameter and 100 feet in height at four levels, (1) at bottom, (2) at height of 25 feet, (3) at height of 50 feet, (4) at height of 75 feet. ($f_s = 18000$).

From Table 2:

At bottom $\frac{H}{D}=\frac{100}{25}=4;\ \frac{A}{D^2}=0.000818 \qquad A=0.511$ sq. in.

At 25′ 0″ $\frac{H}{D}=\frac{75}{25}=3;\ \frac{A}{D^2}=0.000792 \qquad A=0.495$ sq. in.

At 50′ 0″ $\frac{H}{D}=\frac{50}{25}=2;\ \frac{A}{D^2}=0.000720 \qquad A=0.450$ sq. in.

At 75′ 0″ $\frac{H}{D}=\frac{25}{25}=1;\ \frac{A}{D^2}=0.000527 \qquad A=0.329$ sq. in.

From Plate II: As before, follow the vertical line at the right-hand side for diameter 25 feet upward to its intersection with the oblique line representing 100 feet for the height of the grain. From this intersection project horizontally to curve 2 for $f_s = 18000$, then vertically to the line for diameter 25 feet of the set of radial lines for steel. This last intersection will be found to be horizontally opposite a point on the scale for steel areas a little above 0.5 or about 0.51 square inch, which equals area of steel per foot in height at the bottom.

In the same manner by starting in turn with the intersections of the vertical line for 25-foot diameter with the oblique lines for 75 feet[1] height of grain, 50 feet height of grain, 25 feet height of grain, the steel areas may be determined for these levels.

The left-hand section of the plate provides a means of determining the spacing of any size bar there listed to give a required area of steel per foot in height.

Any horizontal line through a required steel area will intersect the oblique line for any chosen size bar on the vertical line giving its required spacing.

For the present example, by reading on the horizontal line through 0.51, it will be found that any of the following combinations of bars and spacings will give the required area of steel at the bottom of the tanks:

$\frac{5}{8}$″ rounds at -- 7″ on center.
$2\frac{1}{2}$″ by $\frac{1}{4}$″ or 1″ by $\frac{5}{16}$″ flats at -------------------- 7″ on center.
2″ by $\frac{3}{16}$″ or $1\frac{1}{2}$″ by $\frac{1}{4}$″ or 1″ by $\frac{3}{8}$″ flats at -------- 9″ on center.
$\frac{5}{8}$″ squares at -- 9″ on center.
$2\frac{1}{4}$″ by $\frac{3}{16}$″ flats at -------------------------------- 10″ on center.
$2\frac{1}{2}$″ by $\frac{3}{16}$″ or $1\frac{1}{2}$″ by $\frac{5}{16}$″ or $1\frac{1}{4}$″ by $\frac{3}{8}$″ flats at --- 11″ on center.

[1] Height of grain equals distance from level of reinforcement to top of grain.

In determining what reinforcing will be used for any tank for its full height we can proceed in three ways to allow for the decrease from bottom to top in the steel area required:

A. Fix a uniform spacing, and use decreasing sizes of bars.

B. Select one size bar and increase its spacing.

C. A combination of the first and second ways.

In any case, having determined the size and spacing of the steel at the bottom, the levels at which the succeeding changes in bar sizes or spacings may be made can be obtained from the chart by methods the reverse of those already outlined. In the present example, if we select $1\frac{1}{2}$ by $\frac{1}{4}$ inch bar, 9 inches on center, as the reinforcing at the bottom, we can proceed as follows:

Follow the diagonal line for $1\frac{1}{2}$ by $\frac{1}{4}$ inch bar down from its intersection with the horizontal line for 0.51 square inch of steel, which gave a spacing of 9 inches, to its intersection with the vertical line for 10-inch spacing; project horizontally from this intersection to radial line 25 for diameters for steel, then vertically to curve 2, again horizontally to the intersection with the vertical line for 25 feet diameter at the right-hand side of the page. This intersection will be found to be on the diagonal line for 50 feet height of grain; 50 feet from the top, therefore, we can change the spacing from 9 inches to 10 inches.

Starting from the intersection of the diagonal line for $1\frac{1}{2}$ by $\frac{1}{4}$ inch bars with the vertical line for 12-inch spacings, we find in the same manner that at the height of 32 feet from the top we can increase the spacing from 10 inches to 12 inches.

We can now either continue to increase the spacing or maintain a constant spacing of 12 inches and change the size of the bar. Selecting 1 by $\frac{1}{4}$ inch as the next size bar, proceed as before from the intersection of its diagonal line with the 12-inch spacing line, and we find that at a distance of 16 feet from the top this size bar can be substituted for the $1\frac{1}{4}$ by $\frac{1}{4}$ inch. In the same manner we will find that 1 by $\frac{1}{8}$ inch bars at 12 inches on center may be used for the last 9 feet.

EXAMPLE 3.

Required: Horizontal reinforcing for the full height in a wall of a concrete bin 10 feet by 10 feet by 60 feet in height ($f_s=16000, f_c=650$, $n=15$, and, for an intermediate span, $m=12$).

From Plate II: L, the unit lateral pressure at the bottom, $=300$ pounds.

From Table 1: For $\frac{H}{D}=\frac{60}{10}=6$, $\frac{L}{D}=29.92$, $L=29.92\times10=300$ pounds.

$$M=\frac{300\times10\times10\times12}{12}=30{,}000 \text{ inch-pounds.}$$

From Table 3a: A wall 6 inches thick with a depth to steel of 5 inches has a resisting moment of 32,300 inch-pounds, requiring 0.462 square inch of steel per foot in height, which is slightly greater than actually required.

Using the formula $a=\frac{L\times l^2}{R}$

we have, $a=\frac{300\times 10\times 10}{70000}=0.43$ square inch.

given by $\frac{5}{8}''$ rounds $8\frac{1}{2}''$ on center.

If it is desirable to have the bars spaced in even inches, we can increase the spacing to 9 inches and investigate to determine the corresponding stresses in steel and concrete.

The area of $\frac{5}{8}''$ rounds 9″ on center equals 0.41 square inch.

$$p=\frac{0.41}{60}=0.0068.$$

for $p=0.0068$ and $n=15$, $k=0.361$, and $j=0.879$.

$$f_s=\frac{M}{a\times j\times d}=\frac{30000}{0.41\times 0.879\times 5}=16640 \text{ pounds.}$$

$$f_c=\frac{2\times f_s\times p}{k}=\frac{2\times 16640\times 0.0068}{0.361}=630 \text{ pounds.}$$

The stress in the concrete is less than the allowable assumed and the stress in the steel is not excessive.

To find the height at which the spacing may be increased to 10 inches: The area of $\frac{5}{8}''$ rounds 10″ on center equals 0.37 square inches.

Using the formula $L=\frac{a\times R}{l^2}$

we have $L=\frac{0.37\times 70000}{10\times 10}=0.37\times 700=259$

$\frac{L}{D}=\frac{259}{10}=25.9$, for which $\frac{H}{D}=2$ (see Table 1).

H, then, is equal to 20, and at a distance of 20 feet from the top we can increase the spacing of the bars to 10 inches.

The area of $\frac{5}{8}''$ rounds 12″ on center equals 0.31 square inch.

$L=0.31\times 700=217$

$\frac{L}{D}=\frac{217}{10}=21.7$, for which $\frac{H}{D}=1.3$.

At a distance of 13 feet from the top we can increase the spacing to 12 inches.

The area of $\frac{1}{2}''$ rounds 12″ on center $=0.20$ square inch.

$L=0.20\times 700=140.$

$\frac{L}{D}=\frac{140}{10}=14$, for which $\frac{H}{D}=0.6$.

At a distance of 6 feet from the top we can change to $\frac{1''}{2}$ rounds 12'' on center.

EXAMPLE 4.

Required: Vertical and lateral pressure at the bottom of a rectangular bin 8 feet by 14 feet by 68 feet in height, walls to be of steel plates, bin to be used for storing barley weighing 40 pounds per cubic foot.

$$\text{Hydraulic radius} = \frac{8 \times 14}{16+28} = 2.55$$

Diameter of equivalent regular bin $= 4 \times 2.55 = 10.2$ feet.

For barley in a steel bin we may select for μ' a value of 0.375 and for k a value of 0.4, then

$$4 \times k \times \mu' = 4 \times 0.4 \times 0.375 = 0.60.$$

$$D' = \frac{10.20}{0.60} = 17 \text{ feet.}$$

At bottom $\frac{H}{D'} = \frac{68}{17} = 4$, for which $\frac{V}{D'} = 49.08$.

$$V = 49.08 \times 17 = 835 \text{ pounds.}$$

For $W = 40$ $V = \frac{40}{50}$ of $835 = 668$ pounds per square foot.

$$L = 0.4 \times 668 = 267 \text{ pounds per square foot.}$$

APPENDIX.

Values of μ', the coefficient of friction of grain on different substances, and of k, the ratio of the lateral to the vertical pressure of grain, as determined by the experiments of different observers are presented herewith.

Coefficients of friction of various kinds of grain on bin walls.

From Airy, Wilfred. The pressure of grain. Institution of civil engineers. Minutes of proceedings. Vol. 131, p. 347-358. London, 1898.

Kind.	Weight of cubic foot loosely filled into measure.	Coefficient of friction.				
		Grain on grain. μ	Grain on rough board. μ'	Grain on smooth board. μ'	Grain on iron. μ'	Grain on cement. μ'
	Pounds.					
Wheat	49	0.466	0.412	0.361	0.414	0.444
Barley	39	.507	.424	.325	.376	.452
Oats	28	.532	.450	.369	.412	.466
Corn	44	.521	.344	.308	.374	.423
Beans	46	.616	.435	.322	.366	.442
Peas	50	.472	.287	.268	.263	.296
Tares	49	.554	.424	.359	.364	.394
Flaxseed	41	.456	.407	.308	.339	.414

Values of μ' and k.

As compiled by Ketchum, M. S., in The design of walls, bins, and grain elevators. N. Y., 1911, p. 346 and 350, from experiments by J. Pleissner.

	μ'		k	
	Wheat.	Rye.	Wheat.	Rye.
Cribbed bin	0.43	0.54	0.4 to 0.5	0.23 to 0.32
Ringed cribbed bin	.58	.78	.4 to .5	.3 to .34
Small plank bin	.25	.37	.34 to .46	.3 to .45
Large plank bin	.45	.55	.3	.23 to .28
Reinforced concrete bin	.71	.85	.30 to .34	.3

Coefficient of friction between wheat and various materials of construction of bin walls.

From Jamieson, J. A. Grain pressures in deep bins. Canadian society of civil engineers. Transactions, 1093. Vol. 17, part 2, pp. 554–607. Montreal, 1905.

Material.	Coefficient of friction. μ'
Wheat on wheat	0.532
Wheat on steel trough-plate bin	.468
Wheat on steel flat plate riveted to tie bars	0.375 to .400
Wheat on steel cylinders, riveted	.365 to .375
Wheat on cement concrete, smooth to rough	.400 to .425
Wheat on tile or brick, smooth to rough	.400 to .425
Wheat on cribbed wooden bins	.420 to .450

k for wheat=0.596.

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